EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Solve : $\sin 5x = \cos 2x$ for all values of x between 0° & 180°.

2. Find the solution set of the equation,

$\log_{\frac{-x^2-6x}{10}} (\sin 3x + \sin x) = \log_{\frac{-x^2-6x}{10}} (\sin 2x)$.

3. Find the value of θ, which satisfy $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$.

4. Find the general solution of the equation, $\sin \pi x + \cos \pi x = 0$. Also find the sum of all solutions in [0, 100].

5. Solve for x, $(-\pi \le x \le \pi)$ the equation ; $2(\cos x + \cos 2x) + \sin 2x (1 + 2\cos x) = 2 \sin x$.

6 Find the range of y such that the equation, $y + \cos x = \sin x$ has a real solution. For $y = 1$, find x such that $0 < x < 2\pi$.

7. Find the general values of θ for which the quadratic function $(\sin\theta)\, x^2 + (2\cos\theta)\, x + \frac{\cos\theta + \sin\theta}{2}$ is the square of a linear function.

8. Find the general solution of the equation, $\tan^2(x + y) + \cot^2(x + y) = 1 - 2x - x^2$.

9. Prove that the equations
(a) $\sin x \cdot \sin 2x \cdot \sin 3x = 1$
(b) $\sin x \cdot \cos 4x \cdot \sin 5x = -1/2$
have no solution

10. Find the general solution of $\sec 4\theta - \sec 2\theta = 2$.

11. Let $f(x) = \sin^6 x + \cos^6 x + k(\sin^4 x + \cos^4 x)$ for some real number k. Determine

(a) all real numbers k for which f(x) is constant for all values of x.

(b) all real numbers k for which there exists a real number 'c' such that f(c) = 0.

(c) If k = –0.7, determine all solutions to the equation f(x) = 0.

12. Solve the equation : $\frac{\sqrt{3}}{2}\sin x - \cos x = \cos^2 x$.

13. Solve : $\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$.

14. Find all the solutions of, $4\cos^2 x \sin x - 2\sin^2 x = 3 \sin x$.

15. If α & β are the roots of the equation, $a\cos\theta + b\sin\theta = c$ then prove that :

(i) $\sin\alpha + \sin\beta = \frac{2bc}{a^2 + b^2}$

(ii) $\sin\alpha \cdot \sin\beta = \frac{c^2 - a^2}{a^2 + b^2}$

(iii) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{2b}{a+c}$

(iv) $\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} = \frac{c-a}{c+a}$

16. Find the general solution of the following equation : $2(\sin x - \cos 2x) - \sin 2x(1 + 2\sin x) + 2\cos x = 0$.

17. Solve the inequality $\sin 2x > \sqrt{2}\sin^2 x + (2 - \sqrt{2})\cos^2 x$.

18. Find the values of x, between 0 & 2π, satisfying the equation ;

$\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$.

19. Solve for x : $\sin 3\alpha = 4\sin\alpha \sin(x + \alpha) \sin(x - \alpha)$ where α is a constant.

20. Find the general solution of the equation $|2\tan x - 1| + |2\cot x - 1| = 2$.

21. Find the set of values of 'a' for which the equation, $\sin^4 x + \cos^4 x + \sin 2x + a = 0$ possesses solutions. Also find the general solution for these values of 'a'.

22. Solve : $\tan^2 2x + \cot^2 2x + 2\tan 2x + 2\cot 2x = 6$.

23. Solve : $\sin^4 x + \cos^4 x - 2\sin^2 x + \frac{3}{4}\sin^2 2x = 0$.

24. Solve :
$\tan^2 x \cdot \tan^2 3x \cdot \tan 4x = \tan^2 x - \tan^2 3x + \tan 4x$.

25. Solve : $\sin^{10} x + \cos^{10} x = \frac{29}{16}\cos^4 2x$.

26. Find the set of values of x satisfying the equality

$\sin\left(x - \frac{\pi}{4}\right) - \cos\left(x + \frac{3\pi}{4}\right) = 1$

and the inequality $\frac{2\cos 7x}{\cos 3 + \sin 3} > 2^{\cos 2x}$.

27. Find the sum of all the roots of the equation , $\sin\sqrt{x} = -1$, which are less than 100 π^2 . Also Find the sum of the square roots of these roots . Now , can we conclude that all the roots $\cos\sqrt{x} = 0$ are also the roots of $\sin\sqrt{x} = -1$? Justify your answer .

28. Solve : $\sin\left(\frac{\sqrt{x}}{2}\right) + \cos\left(\frac{\sqrt{x}}{2}\right) = \sqrt{2}\sin\sqrt{x}$.

29. Find the general solution of the equation,

$\sin\frac{2x+1}{x} + \sin\frac{2x+1}{3x} - 3\cos^2\frac{2x+1}{3x} = 0$

30. Solve the equation : $\sin 5x = 16\sin^5 x$.

31. Solve for x & y : $\begin{matrix} x\cos^3 y + 3x\cos y\sin^2 y = 14 \\ x\sin^3 y + 3x\cos^2 y\sin y = 13 \end{matrix}$

32. Solve the equation : $\cot x - 2\sin 2x = 1$.